① a) $x^2 - y^2 = 2xy$

$$2x - 2y\frac{dy}{dx} = 2x \cdot \frac{dy}{dx} + y \cdot 2$$
$$2x - 2y = 2x\frac{dy}{dx} + 2y\frac{dy}{dx}$$
$$2x - 2y = \frac{dy}{dx}(2x + 2y)$$
$$\frac{2x-2y}{2x+2y} = \frac{dy}{dx}$$

b) $x^3 + xy + y^3 = 4$

$$3x^2 + x\frac{dy}{dx} + y \cdot 1 + 3y^2 \cdot \frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{-y - 3x^2}{x + 3y^2}$$

c) $y^{-1} + x^{-1} = 2$

$$-y^{-2} \cdot \frac{dy}{dx} + -x^{-2} = 0$$
$$-\frac{1}{y^2} \cdot \frac{dy}{dx} - \frac{1}{x^2} = 0$$
$$-\frac{1}{y^2} \cdot \frac{dy}{dx} = \frac{1}{x^2}$$
$$\frac{dy}{dx} = -\frac{y^2}{x^2}$$

d) $3x^4 = (2xy - 1)^3$

$$12x^3 = 3(2xy-1)^2 \cdot \left(2x\frac{dy}{dx} + y \cdot 2\right)$$
$$\frac{12x^3 - 6y(2xy-1)^2}{6x(2xy-1)^2} = \frac{dy}{dx}$$

② a) $y^2 = \frac{x^2 - 4}{x^2 + 4}$

$$2y\frac{dy}{dx} = \frac{(x^2+4)(2x) - (x^2-4)(2x)}{(x^2+4)^2}$$
$$\frac{dy}{dx} = \frac{(x^2+4)(2x) - (x^2-4)(2x)}{(x^2+4)^2 \cdot 2y}$$
$$\left.\frac{dy}{dx}\right|_{(2,0)} = \frac{8 \cdot 4 - 0}{8^2 \cdot 0} \text{ undef.}$$

Tangent: $x = 2$

Normal: $y = 0$

b) $(x+y)^3 = x^3 + y^3$

$$3(x+y)^2\left(1 + \frac{dy}{dx}\right) = 3x^2 + 3y^2 \cdot \frac{dy}{dx}$$

plug in $(-1,1)$ $\longrightarrow$ $3(0)^2 \cdot \left(1 + \frac{dy}{dx}\right) = 3(-1)^2 + 3(1)^2 \cdot \frac{dy}{dx}$

$$\frac{dy}{dx} = -1$$

normal ~~tangent~~: $y - 1 = x + 1$

tangent ~~Normal~~: $y - 1 = -1(x+1)$

(3) $4y^3 \cdot \frac{dy}{dx} = 2y\frac{dy}{dx} - 2x$

$\frac{dy}{dx} = \frac{2x}{2y-4y^3}$

horiz tangent when $\frac{dy}{dx} = 0$

$2x = 0 \Rightarrow x = 0$

when $x=0 \to y^4 = y^2$

$y^4 - y^2 = 0$

$y^2(y^2-1) = 0$

$y=0 \,;\, y = \pm 1$

horiz @ $(0,-1)$, $\cancel{(0,0)}$, $(0,1)$

Vert. tangent $\frac{dy}{dx}$ undef

$2y - 4y^3 = 0$

$2y(1-2y^2) = 0$

$y = 0 \quad y = \pm\frac{1}{\sqrt{2}}$

$\downarrow$

$\cancel{(0,0)}$

when $y = \pm\frac{1}{\sqrt{2}} \to \frac{1}{4} = \frac{1}{2} - x^2$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$

So vertical @ $\left(\pm\frac{1}{2}, \pm\frac{1}{\sqrt{2}}\right)$ (4 points)

@ $(0,0)$ $4y^3$ approaches 0 much faster than $2x$ or $2y$ so

$\frac{dy}{dx} \to \frac{2x}{2y} = \pm\frac{2}{2} = \pm 1$ (b/c graph approaches $y^2 = x^2 \to y = \pm x$ near 0)

So graph is neither horiz. or vert @ $(0,0)$

(4) a) $y = x^4 - 3x^2 + 4$

$y' = 4x^3 - 6x$

$y' = 0 \rightarrow 2x(2x^2-3) = 0$

$x = 0$ | $x = \pm\sqrt{\frac{3}{2}}$ outside domain

$y(-1) = 2$ Abs. min

$y(0) = 4$ Abs. max

$y(1) = 2$

b) $y = x^3 + 6x^2 + 9x + 3$

$y' = 3x^2 + 12x + 9$

$y' = 0 \rightarrow 3(x^2 + 4x + 3) = 0$

$x = -1, -3$

$y(-4) = -64 + 96 - 36 + 3 = -1$

$y(-3) = -27 + 54 - 27 + 3 = 3$

$y(-1) = -1 + 6 - 9 + 3 = -1$

$y(0) = 3$

Abs min: -1, Abs Max 3

(5) a) $y = x^4 - 10x^2 + 9$

$y' = 4x^3 - 20x = 4x(x^2 - 5)$

$x = 0 \quad x = \pm\sqrt{5}$

y': $-$ | $+$ | $-$ | $+$ at $-\sqrt{5}$, 0, $\sqrt{5}$

y increasing for $x \in (-\sqrt{5}, 0) \cup (\sqrt{5}, \infty)$

y decreasing for $x \in (-\infty, -\sqrt{5}) \cup (0, \sqrt{5})$

b) $y = \frac{-x}{x^2+4}$

$y' = \frac{(x^2+4)(-1) - (-x)(2x)}{(x^2+4)^2}$

$y' = \frac{-x^2 - 4 + 2x^2}{(x^2+4)^2} = \frac{x^2-4}{(x^2+4)^2}$

$y' = 0$ when $x = \pm 2$

y' undef $\emptyset$

y': $+$ | $-$ | $+$ at -2, 2

y increasing on $x \in (-\infty, -2) \cup (2, \infty)$

y decreasing for $(-2, 2)$